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ESTIMATES OF RUPTURE LIFE-CONSTANT LOAD

F. A. Leckie and W. Wojewodzki
Department of Engineering, University of Leicester, Leicester LE1 7RH, England

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1. INTRODUCTION

The life of components operating at elevated temperatures are often limited by a material deterioration referred to as creep rupture. This deterioration takes the form of voids and fissures which grow on grain boundaries and is associated with the tertiary portion of the constant stress creep test when the creep strain rate increases. The tertiary behaviour can also be explained in terms of a stress which increases as the cross-sectional area decreases. This explanation however[1], is normally associated with relatively high stress levels, and since the guarantee of prolonged life implies low working stresses of about 25% of the yield stress at temperature, it is material deterioration rather than geometric effects which influence the life of a component.

At prsent it is normal to estimate the rupture life of a component by using the maximum stress obtained from a steady state analysis in conjunction with the stress-time rupture curve of the material[1]. This procedure which has a certain intuitive appeal does not take account of the stress distribution effects which must take place as a result of the material softening in the tertiary region. The importance of stress redistribution has been demonstrated in experiments by Leckie and Hayhurst[2] on plates with stress raisers. More complex calculations which take into account the effects of damage and stress distribution have been performed by Hayhurst[3] and Hayhurst and Leckie[4]. These calculations were made on rather simple components initially, but more recently Hayhurst, Dimmer and Chernuka[5] have extended the procedures for use with finite elements so that more complex components may be analysed. Unfortunately the computing power required to analyse these problems is very great and it is likely that such procedures will only be used as part of a final design assessment. There is an incentive therefore to seek general results of the type which require relatively modest calculations which have proved so useful when studying time dependent creep deformations.

Attempts in this dirction have been made by Martin and Leckie[6] who were able to obtain lower bound expressions for the rupture time of kinematically determinate structures. The results were further generalised by Leckie and Hayhurst[2] who derived formulae which take into consideration both the effects of stress redistribution and multi-axial states of stress, but which are still limited to kinematically determinate structures. In spite of this limitation experimental results on a variety of components, not all of which were kinematically determinate, agreed well with theoretical predictions. The problem has also been studied by Goodall and Cockroft[7] who obtained an upper bound on the rupture life by optimising a functional given in terms of the multi-axial stress isochronous rupture surface.

In this and subsequent reports a study is made of some approximate methods of calculating the rupture times of structures. This paper is concerned with structures which operate at constant temperature and are subjected to constant load.

The advantages to be gained from describing the creep deformation behaviour of structures in terms of a Representative Deformation Stress are well known, and as far as possible attempts are made to express the results of the present paper in terms of a Representative Rupture Stress.

2. THE CONSTITUTIVE EQUATIONS

The strain rate $\dot{\epsilon}_{ij}$ is assumed to be the sum of the elastic $\dot{e}_{ij}$ and creep $\dot{v}_{ij}$ strain rate components, so that

$$\dot{\epsilon}_{ij} = \dot{e}_{ij} + \dot{v}_{ij} \tag{2.1}$$

The elastic strains e_{ij} are related to the stress σ_{ij} by the relation

$$e_{ij} = C_{ijkl}\sigma_{kl} \tag{2.2}$$

where C_{ijkl} is assumed to be constant.

The expressions used to describe the creep strain rate are those derived by Leckie and Hayhurst[2] which are a generalisation of stress first proposed by Kachanov[8] suitably modified to fit the results of multi-axial stress experiments. By introducing an internal state variable ψ Kachanov was able to modify the Norton Creep Law so that the tertiary portion of the creep curve could be represented. In the Kachanov equations the functions were selected to fit creep data and there was no apparent attempt made to interpret the physical meaning of ψ other than in vague terms of material damage. On the basis of the studies reported by metal physicists Leckie and Hayhurst[2] suggested however that the ψ parameter can properly be used to measure the physical processes of deterioration.

The constitutive equation for the creep rate $\dot{v}_{ij}$ is

$$\dot{v}_{ij}/\dot{v}_0 = \frac{1}{\psi^n}\,\phi^n(\sigma_{ij}/\sigma_0)\partial\phi/\partial(\sigma_{ij}/\sigma_0). \tag{2.3a}$$

In this expression ϕ is homogeneous function of degree one and is equal to unity when the applied stress is uniaxial and of magnitude σ_0. The material constants are $\dot{v}_0$, σ_0 and n. When the material is undamaged $\psi = 1$ and the expression for $\dot{v}_{ij}$ reduces to that suggested by Calladine and Drucker[9]. As damage occurs ψ decreases so that the strain rates increase, but the form of (2.3a) implies that the ratio of strain rate components remains constant, which is apparently in accord wih experimental observations Leckie and Hayhurst[2].

The damage rate equation has the form

$$\dot{\psi} = -\frac{A}{\psi^\nu}\,\Delta^\nu(\sigma_{ij}/\sigma_0) \tag{2.3b}$$

where A and ν are material constants and Δ is a homogeneous function of degree one in (σ_{ij}/σ_0).

Integrating this equation and using the rupture condition $\psi = 0$ gives the expression for rupture time t_R as

$$t_R = 1/A(1+\nu)\Delta^\nu(\sigma_{ij}/\sigma_0) \tag{2.4}$$

The vaues of A and ν can be determined from uniaxial tests. Isochronous surfaces are given by $\Delta(\sigma_{ij}/\sigma_0) = \text{constant}$. The function $\Delta(\sigma_{ij}/\sigma_0)$ can take various forms. Multi-axial rupture experiments show the isochronous rupture surface of some important materials, such as stainless steel and aluminium, satisfy a Huber-Mises shear criterion. Other metals such as copper satisfy a maximum stress criterion (Fig. 1). The rupture criterion for most metals Hayhurst[10] apparently lie between these two extremes. It is convenient to refer to those metals whose constant energy dissipation and isochronous surfaces have the same form as ϕ/ϕ materials. The costitutive equations have the form

$$\begin{aligned}\dot{v}_{ij}/\dot{v}_0 &= \frac{1}{\psi^n}\,\phi^n(\sigma_{ij}/\sigma_0)\partial\phi/\partial(\sigma_{ij}/\sigma_0)\\ \dot{\psi} &= -\frac{A}{\psi^n}\,\phi^\nu(\sigma_{ij}/\sigma_0)\end{aligned} \tag{2.5a,b}$$

Materials which obey the constitutive eqn (2.3a,b) are referred to as ϕ/Δ.

3. AN UPPER BOUND ON RUPTURE TIME

A body of volume V (Fig. 2a) subjected to constant loads P_i, consists of material whose constitutive equations are those described in Section 2. After the initial elastic response,

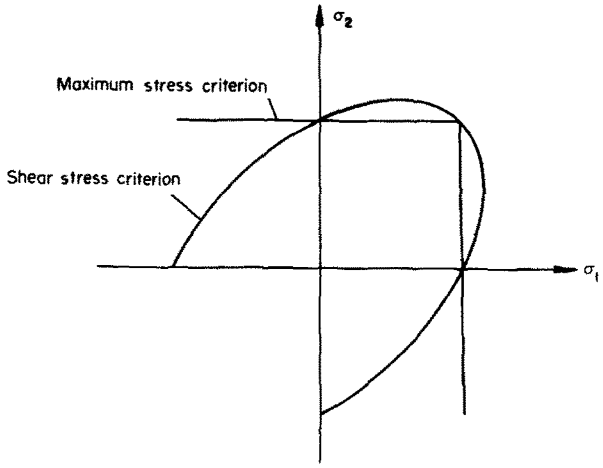

Fig. 1. Isochronous surfaces.

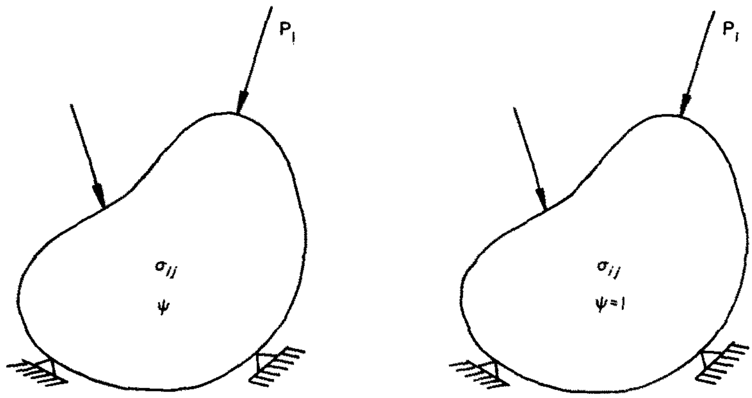

Fig. 2. (a) Actual component; (b) comparison component.

interaction between the elastic and creep strains results in stress redistributions. Further stress redistribution can be expected however as a result of the softening resulting from material deterioration. At time t_1 local rupture will occur at some point in the body when the value of ψ at that point is zero. A damage front will then spread out from this point when only a portion $\bar{V}$ of the body can sustain stress. Final rupture occurs at time t_R when the body cannot sustain the applied load.

At time $t < t_1$ let the stress and damage dstributions be represented by σ_{ij} and ψ respectively. Equation (2.3b) can be expressed in the form

$$-\frac{\quad}{(1+\nu)A}\frac{\mathrm{d}}{\mathrm{d}t}\psi^{\nu+1} = \Delta^{\nu}(\sigma_{ij}/\sigma_0) \tag{3.1}$$

and integrating over the volume V gives the relationship

$$-\frac{1}{(1+\nu)A}\frac{\mathrm{d}}{\mathrm{d}t}\int_V \psi^{\nu+1}\,\mathrm{d}V = \int_V \Delta^{\nu}(\sigma_{ij}/\sigma_0)\,\mathrm{d}V \tag{3.2}$$

Consider now an imaginary body identical in shape to that under consideration and subjected to the same loads P_i (Fig. 2b). The material of the body is an elastic creeping material with constitutive equations.

$$e_{ij} = C_{ijkl}\sigma_{kl}$$

$$\dot{v}_{ij}/\dot{v}_0 = \Delta^{\nu-1}(\sigma_{ij}/\sigma_0)\partial\Delta/\partial(\sigma_{ij}/\sigma_0). \tag{3.3.a,b}$$

It should be noted that this material does not suffer material deterioration. The creep energy dissipation rate is

$$\dot{D}/\sigma_0\dot{v}_0 = \Delta^{\nu}(\sigma_{ij}/\sigma_0)$$

so that the surfaces of constant energy dissipation rate are given by $\Delta(\sigma_{ij}/\sigma_0) = \text{constant}$ and coincide with the isochronous surface of the rupturing material. In the analysis which follows it is necessary to assume that the isochronous surfaces are convex so that use can be made of certain established relationships. According to Hayhurst[10] existing experimental evidence supports the validity of the convexity assumption.

Suppose now that $\bar{\sigma}_{ij}$ is the steady state solution for the imaginary body. From the complementary energy theorem

$$\int_V \Delta^{\nu}(\bar{\sigma}_{ij}/\sigma_0)\,\mathrm{d}V \leqslant \int_V \Delta^{\nu}(\sigma_{ij}/\sigma_0)\,\mathrm{d}V.$$

Using this result in the global damage rate eqn (3.2) gives the inequality

$$-\frac{1}{(1+\nu)A}\frac{\mathrm{d}}{\mathrm{d}t}\int_V \psi^{\nu+1}\,\mathrm{d}V = \int_V \Delta^{\nu}(\sigma_{ij}/\sigma_0)\,\mathrm{d}V \geqslant \int_V \Delta^{\nu}(\bar{\sigma}_{ij}/\sigma_0)\,\mathrm{d}V \tag{3.4}$$

When $t > t_1$, then $\psi = 0$ for that part of the structure which has ruptured. Consequently

$$\int_{\bar{V}} \psi^{\nu+1}\,\mathrm{d}V = \int_V \psi^{\nu+1}\,\mathrm{d}V$$

so that the inequality (3.4) is still satisfied as the rupture surface moves through the body. Replacing the inequality by an equality and using the condition $\int_V \psi^{\nu+1}\,\mathrm{d}V = 0$ at rupture times gives a time to rupture which is an upper bound on the rupture time t_R. Hence

$$t_R < t_u = \frac{V}{A(1+\nu)\int_V \Delta^{\nu}(\bar{\sigma}_{ij}/\sigma_0)\,\mathrm{d}V}. \tag{3.5a}$$

Using this result together with the expression for the time to rupture of a uniaxial specimen gives a Representative Rupture Stress σ_u

$$\sigma_u/\sigma_0 = \left\{\int_V \Delta^{\nu}(\bar{\sigma}_{ij}/\sigma_0)\,\mathrm{d}V/V\right\}^{1/\nu}. \tag{3.6}$$

The representative deformation stress σ_D has been found to be useful in defining creep deformation, since the corresponding uniaxial strain gives a measure of the average strain in the component. The definition for a material with creep index n is

$$\sigma_D/\sigma_0 = \left\{\int_V \phi^{n+1}(\bar{\sigma}_{ij}/\sigma_0)\,\mathrm{d}V/V\right\}^{1/(n+1)}. \tag{3.7}$$

Since in many structures the representative deformation stress is almost independent of n [11] it is to be expected therefore that for structures of ϕ/ϕ materials the Representative Deformation and Upper Bound Rupture Stress σ_D and σ_u will not greatly differ.

4. LOWER BOUNDS OF RUPTURE LIFE

Attempts to obtain a general expression for a lower bound on rupture life have not been successful and to date only certain special results have been obtained. It is usually assumed that the Odqvist [1] method gives a lower bound and an attempt, of limited success, to justify the procedure is described in this section.

Following the method proposed by Martin and Leckie [6], Leckie and Hayhurst [2] were able to obtain a lower bound on the rupture time of kinematically determinate components which takes into consideration the effects of multi-axial stress distributions. For completeness and for convenience the procedure is briefly outlined. Finally, in this section a rupture time expression is suggested which is given in terms of the elastic stress distribution, and which may be useful in estimating ruptur life when only the elastic solution is available. Unfortunately, it has not been possible to prove the general validity of the result, but it may be useful if applied with caution.

4.1 *The Odqvist method*

In this method the maximum steady state stress component is used in conjunction with the uniaxial stress-time to rupture graph, to give an estimate of the time for first rupture to appear. Allowance can be made to include the influence of multi-axial stress states on the rupture life, but the method does not take into consideration the beneficial effects of stress redistribution resulting from material deterioration. However, for materials which show no teriary effects whose creep curves have the form shown in Fig. 3 then the Odqvist method gives the exact time to first rupture. Intuition suggests that the method is likely to give a lower bound on rupture time but apparently no formal proof of the validity of the procedure exists. In this section an attempt is made to provide such a proof.

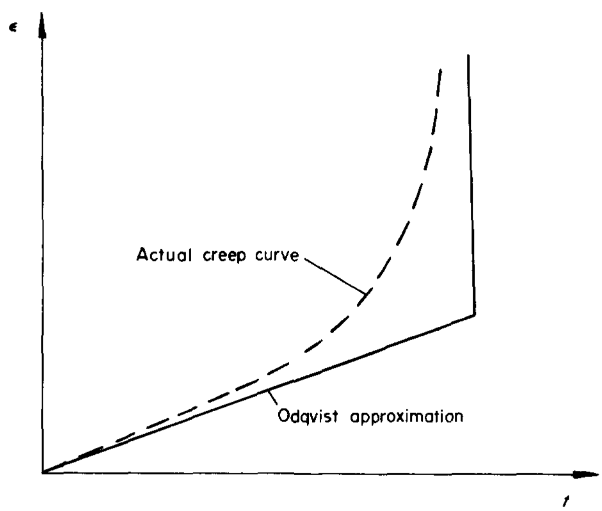

Fig. 3. Creep curve.

It is necessary to neglect the effects of elastic strains so that after the application of the load at time $t = 0^*$, the stress distribution is that given by the steady state solution σ^s_{ij} when ψ is everywhere unity. Since the elastic effects are excluded the stress distribution σ_{ij} at time t is such that the total energy dissipation rate is a minimum. Using the results of eqn (2.3) the total energy dissipation rate is

$$\int_A \frac{P_i \dot{u}_i}{\sigma_0 \dot{v}_0} \mathrm{d}A = \int \sigma_{ij}\dot{v}_{ij} \,\mathrm{d}V/\sigma_0\dot{v}_0 = \int \dot{D} \,\mathrm{d}V/\sigma_0\dot{v}_0 = \int_V \psi\phi^{n+1}(\sigma_{ij}/\psi\sigma_0)\,\mathrm{d}V. \tag{4.1}$$

The damage rate eqn (2.3b) can be written in the form

$$-\frac{\mathrm{d}\psi^2}{\mathrm{d}t} = 2A\psi\Delta^{\nu}(\sigma_{ij}/\psi\sigma_0).$$

For the special case of a ϕ/ϕ material with $\nu = n+1$ this equation can be written in the form

$$-\frac{1}{2A}\frac{d\psi^2}{dt} = \psi\phi^{n+1}(\sigma_{ij}/\psi\sigma_0).$$

Integrating over the volume

$$-\frac{1}{2A}\frac{d}{dt}\int_V \psi^2\,dV = \int_V \psi\phi^{n+1}(\sigma_{ij}/\psi\sigma_0)\,dV \leq \int_V \psi\phi^{n+1}(\sigma^s_{ij}/\psi\sigma_0)\,dV$$

where σ^s_{ij} is the steady state solution in the undamaged structure. Hence for any given distribution of ψ the rate of decrease $\int \psi^2\,dV$ [defined in (4.1)] is less than that associated with the stress distribution σ^s_{ij}. Also $(d/dt)\int_V \psi^2\,dV$ becomes infinitely large when ψ at one point approaches zero. If therefore σ^s_{ij} is used as the stress distribution then a lower bound on rupture time is obtained by integrating the equation

$$-\frac{1}{A}\frac{d\psi}{dt} = \phi_m{}^{n+1}(\sigma^s_{ij}/\sigma_0) \tag{4.2}$$

where ϕ_m is the maximum value of ϕ for the stress distribution σ^s_{ij}. This corresponds to the Odqvist method for which the rupture time is

$$t_R = 1/A(1+n)\phi_m{}^{n+1}(\sigma^s_{ij}/\sigma_0). \tag{4.3}$$

Unfortunately it does not appear possible in general to extend this proof to deal with other values of ν or for ϕ/Δ materials. A small extension of the proof is possible for ϕ/Δ materials in a state of plane stress. In plane stress it is to be expected that

$$\Delta(\sigma_{ij}/\sigma_0) \leq \phi(\sigma_{ij}/\sigma_0).$$

In these circumstances it is easy to show that the result (4.3) again holds.

4.2 *Lower bound for kinematically determinate components*

If elastic effects are neglected it is possible in certain circumstances to determine a lower bound on the rupture time of kinematically determinate components.

For ϕ/ϕ materials the expression given by Leckie and Hayhurst[2] for the lower bound rupture time t_R is

$$t_R = \frac{\displaystyle\int_V (\dot{D}_s/\sigma_0\dot{v}_0)\,dV}{\displaystyle(1+\nu)A\int_V (\dot{D}_s/\sigma_0\dot{v}_0)^{(n+1+\nu)/(n+1)}\,dV}. \tag{4.4a}$$

In this expression $\dot{D}_s = \sigma_0\dot{v}_0\phi^{n+1}(\sigma^s_{ij}/\sigma_0)$ is the creep energy dissipation rate corresponding to the steady state stress distribution σ^s_{ij} when $\psi = 1$.

When expressed in terms of a Representative Rupture Stress σ_R the result is

$$\sigma_R/\sigma_0 = \left\{\frac{\displaystyle\int_V (\dot{D}_s/\sigma_0\dot{v}_0)\,dV}{\displaystyle\int_V (\dot{D}_s/\sigma_0\dot{v}_0)^{(n+1+\nu)/n+1}\,dV}\right\}^{1/\nu} \tag{4.4b}$$

A corresponding formula for ϕ/Δ materials cannot generally be found and additional calculations are normally necessary. For plane stress conditions however the following expression for a lower bound on rupture time can be derived

$$t_R = \frac{\int_V (\dot{D}_s/\sigma_0\dot{v}_0)\,\mathrm{d}V}{(1+\nu)A\int (\dot{D}_s/\sigma_0\dot{v}_0)\Delta^\nu(\sigma^s_{ij}/\sigma_0)\,\mathrm{d}V} \tag{4.5a}$$

which gives the representative rupture stress

$$\sigma_R/\sigma_0 = \left\{\frac{\int_V (\dot{D}_s/\sigma_0\dot{v}_0)\,\mathrm{d}V}{\int_V (\dot{D}_s/\sigma_0\dot{v}_0)\Delta^\nu(\sigma^s_{ij}/\sigma_0)\,\mathrm{d}V}\right\}^{1/\nu}. \tag{4.5b}$$

While the above expressions are only valid for kinematically determinate components, it has been proposed that they may reasonably be applied to other components by making the assumption that the pattern of deformation during the rupture process remains identical to the steady state pattern. Experimental results on a number of components [2, 12] suggest that such an assumption is justifiable.

4.3 *A possible lower bound*

In Section 3 the rate of change of the global quantity $\int_V \psi^{\nu+1}\,\mathrm{d}V$ was found to be proportional to $\int_V \Delta^\nu(\sigma_{ij}/\sigma_0)$ (eqn 3.2) where σ_{ij} is the current state of stress. A lower bound on this rate was derived by determining the steady state solution corresponding to the constitutive eqns (3.3). A corresponding upper bound of this rate cannot in general be found, but on the basis of heuristic argument it may be possible to make some progress.

Contours of constant values of the damage rate integral $\int_V \Delta^\nu(\sigma_{ij}/\sigma_0)\,\mathrm{d}V$ are shown in Fig. 4. The minimum value of this integral is given by the stress distribution $\bar{\sigma}_{ij}$ (defined in Section 3),

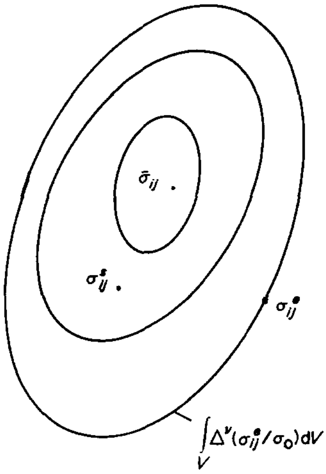

Fig. 4. Contours of global damage rate function $\int_V \Delta^\nu(\sigma_{ij}/\sigma_0)\,\mathrm{d}V$.

and it is this value which is used to obtain an upper bound on rupture time. The state of stress in the body at time $t = 0^*$ when the load is first applied is the elastic stress distribution σ^e_{ij} and the corresponding damage rate integral is $\int_V \Delta^\nu(\sigma^e_{ij}/\sigma_0)\,\mathrm{d}V$. It was shown by Leckie and Martin [13] that for undamaged structures the stress distribution changes monotonically from the initial elastic stress σ^e_{ij} to the steady state stress σ^s_{ij}. Consequently, initially, before damage has occurred in the structure, a similar path is followed by the stress distribution. If the stress state σ^s_{ij} lies within the contour $\int_V \Delta^\nu(\sigma^e_{ij}/\sigma_0)\,\mathrm{d}V$ then the global damage rate (eqn 3.2) will initially decrease. If, on the other hand, σ^s_{ij} lies outside the contour then the initial global damage rate will increase. When damage takes place the corresponding steady state stress distribution will no longer coincide with σ^s_{ij}. Let it be assumed however that the steady state stress distribution continues to lie within or outside the contour $\int_V \Delta^\nu(\sigma^e_{ij}/\sigma_0)\,\mathrm{d}V$. If σ^s_{ij} stress distribution lies within the contour and the above assumptions are made, then the lower bound on the rupture time is (eqn 3.5)

$$t_R > t_L = \frac{V}{A(1+\nu)\int_V \Delta^\nu(\sigma^e_{ij}/\sigma_0)\,\mathrm{d}V} \tag{4.6a}$$

the corresponding representative stress being

$$\sigma_L/\sigma_0 = \left\{\int_V \Delta^\nu(\sigma^e_{ij}/\sigma_0)\,\mathrm{d}V/V\right\}^{1/\nu}. \tag{4.6b}$$

If, on the other hand the steady state stress σ^s_{ij} lies outside the contour $\int_V \Delta^\nu(\sigma^e_{ij}/\sigma_0)\,\mathrm{d}V$ as indicated in Fig. 5, then a more appropriate expression for the rupture time is

$$t_L = \frac{V}{A(1+\nu)\int_V \Delta^\nu(\sigma^s_{ij}/\sigma_0)\,\mathrm{d}V} \tag{4.7a}$$

with the corresponding representative stress

$$\sigma_L/\sigma_0 = \left\{\int_V \Delta^\nu(\sigma^s_{ij}/\sigma_0)\,\mathrm{d}V/V\right\}^{1/\nu}. \tag{4.7b}$$

These expressions are easy to compute from the elastic and steady state stress distributions and clearly it is the higher value of σ_L which should be selected.

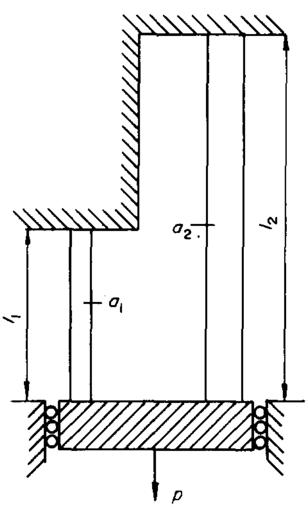

Fig. 5. Two bar structure.

5. EXAMPLE

In order to provide a base for judging the effectiveness of the bounding techniques the two bar structure shown in Fig. 5 was analysed by integrating the appropriate rate equations numerically. For the particular geometry considered $l_2/l_1 = 2{\cdot}5$ and $a_2/a_1 = 2$ giving a stress concentration factor of 2·5 which is the greatest value normally allowed in design. The values selected for the constants and load level are $n = 3$, $\nu = 4$, $\dot{v}_0 E/A\sigma_0 = 1000$, $P/(a_1 + a_2)\sigma_0 = 1{\cdot}2$, $A = 1/438000$ which give results typical of those occurring in practice.

It can be seen from Table 1 that the lower bound (4.4) gives a failure time between first and final failure and is the bound which gives the best result. The worse bound (4.6) which makes use of the elastic solution is conservative in the measure of representative stress by 9%. Even this worst bound lies within the scatter normally associated with creep rupture data and should

Table 1.

	Lower bound [elastic (4.6)]	Lower bound (4.4)	Exact solution 1st Bar	Exact solution 2nd Bar	Upper bound [steady state (3.5)]
Failure time (hour)	24833	33704	33036	34552	40469
Representative rupture stress (σ/σ_0)	1·37	1.·27	1·275	1·263	1·21

consequently be of help at the initial stage of design when only elastic solutions are likely to be available.

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